Using interval analysis in real-time for mobile robot integrity monitoring

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Localization Quality of Service

3 key attributes

Availability
- The percentage of time that the localizer is providing estimates respecting navigation requirements

Accuracy
- The statistical difference between the location estimate and the true value of the location.

Integrity
- A measure of the trust that can be put in the information from the localizer (i.e., likelihood of undetected failures).
Integrity

Ability to associate to a result a reliable indication of confidence.

Civil aviation

- The indicator is a confidence domain
- Alert Limit: maximum tolerable error in the position solution
- Time to alert: maximum time between the occurrence of an alarm condition and its signaling
- Integrity risk: probability that the position error exceeds the alert limit without the user being informed during the time to alert

[RTCA/DO-229D]
Localization uncertainty

- A localization system implements measures on exteroceptive landmarks.
- The design of a system (coverage, geometry, signals, landmarks...) is made in terms of quality of service objectives.
- In complex environment, the visibility of the landmarks and the quality of the measures can be severely degraded.
- The user must estimate in real time the confidence in the estimated position.
Localization uncertainty

Knowledge of positioning uncertainty is necessary to decide if position information is relevant for the current application

Localization uncertainty is time and location dependent
- Geometrical configuration - Noise - Faults

Second order moments
You are most probably here...

...and less probably here

Protection levels
You are somewhere inside this box
On the use of the confidence during navigation

Confidence must be compatible with navigation requirements

- Landing of air plan
- Road navigation
**Bounded-error framework**

A way to compute positioning confidence domains is to use interval analysis.

The set of positions compatible with the measurements and constraints:
- Arbitrary shaped solution set
- Disconnected sets in case of ambiguity
- With a integrity risk computed using the pdf of the noise
Outline

Sources of information
Location domain computation using set inversion
Adding maps for challenging environments
Robust set inversion
Risk computation
Confidence domain validation through real experiments
Fault detection and identification
Conclusion
Sources of information
Classical GPS positioning problem

- Receiver measures pseudo-ranges: range + offset

- 4 unknowns: \(x, y, z, dtu\)

-Pseudo-range observation model:

\[
\begin{align*}
\rho_1 &= \sqrt{(x - x_{s1})^2 + (y - y_{s1})^2 + (z - z_{s1})^2 + c \cdot dtu} \\
\rho_2 &= \sqrt{(x - x_{s2})^2 + (y - y_{s2})^2 + (z - z_{s2})^2 + c \cdot dtu} \\
&\quad \ldots \\
\rho_p &= \sqrt{(x - x_{sp})^2 + (y - y_{sp})^2 + (z - z_{sp})^2 + c \cdot dtu}
\end{align*}
\]

\(x_{si}, y_{si}, z_{si}\) are satellite positions (broadcast)
\(\rho_i\) are corrected pseudoranges:
Proprioceptive sensors

- Easily accessible via the CAN-bus of modern vehicles
- Used to determine the movement of the vehicle
Digital Terrain Model

Digital representation of the altitude

- Square mesh (metric)
- pseudo-square (meridian-parallel)

Examples:
- SRTM-3 (NASA) : World
  - Mesh 90 m / ±14 m alti
- MNT BD Topo (IGN) :
  - France
  - Mesh 25 m / ±1 m alti
Map

Linear representation
- Poly-lines describing the road network
  - attributes (speed limit, altitude, etc.)

[Betaille et al, 2008] Making an enhanced map for lane location based services

Surface
- Surface Describing the drivable space
  - 3D points
  - Triangular facets

[Paparoditis et al, 2000] Surface reconstruction in urban areas from multiple views of aerial digital frame cameras
3D facets of the drivable space

Produced by the French Institut Géographique National
- Photogrammetry from aerial photographs
- Precision of vertices
  - 5 cm planar / 20 cm altitude
Location domain computation using set inversion
Bounded-error GPS positioning

Bounded-error framework
- Measurements = Intervals
- Intervals are assumed to include the true value with a given probability

Positioning is a Constraint Satisfaction Problem
- Measurements = Constraints on position
- Position = Intersection of constraints
Pseudorange constraint

Each measurement is a constraint on position

\[ [\rho_i] = \sqrt{(e - e^s)^2 + (n - n^s)^2 + (u - u^s)^2 + [d]} \]

Prior position box is contracted with « fall - climb » constraint propagation

The domains of the variables are narrowed without losing solution

Contraction is successively applied with each pseudo-range, until a fixed point
Illustrative example

constraint from B1
range measurement

prior position box

constraint from B2

constraint from B3
constraint from B1
range measurement
constraint from B2
constraint from B3
constraint from B1
range measurement

constraint from B2

constraint from B3
constraint from B1
range measurement

constraint from B2

constraint from B3
constraint from B1
range measurement

contracted position box

constraint from B2
constraint from B3
Refining the solution set: Subpavings

Boxes only provide a rough approximation
Better approximation of arbitrary sets: subpavings
Adding maps for challenging environments
Map: 3D facets constraint

Bounded error framework allows taking into account surface constraint

1 facet constraint
- Vertices coordinates are boxes (uncertainty)
- Facet plane constraint
- 3 facet edges constraints
Drivable space constraint

- Union of facet constraints
Facet selection (map matching)

Use topology to mark eligible neighbors from previous epoch facets set.

- Speeds up computation
- Limits ambiguous solutions in poor GPS conditions and dense road networks
Positioning algorithm

Rough Prediction with odometry

Facet selection
- prior box
- candidate facets

SIVIA
- Pick a box
- GPS contraction
- Map contraction
- Bisection

$k \leftarrow k + 1$

solution subpaving
Integrity monitoring
Integrity risk

The risk is due to GPS pseudo-range outliers
- e.g. NLOS multi-path
- Measurements that don’t respect the bounded error model
Consequences of erroneous measurements

Intersection of all the constraints is not robust
q-relaxed Intersection

\[
\bigcap X_i = \bigcap X_i
\]
Robustness to wrong measurements

Robustness: Intersection of at least \( m-q \) constraints.

\[ \rightarrow \text{q-relaxed intersection} \]
1-relaxed intersection
Risk computation
Confidence domain characterization

How to compute a risk associated with a computed domain?

knowing that the noise has not a bounded support?
Risk computation

One measurement

Probability of having the measurement inside the bounds

\[
p = \Pr(y \in [y_{\text{meas}}]) = \int_{e_y}^{e_y} f_{e_y}(\alpha) d\alpha.
\]

“m” measurements

Probability of having \( m \) measurement inside the bounds

Independence assumption of measurement noise

\[
\Pr(n_{\text{ok}} = m) = p^m.
\]
Risk computation using robust set inversion

The result is guaranteed as long as there are “m-q” intervals including the true value (i.e. no more than q outliers)

Probability of having $k$ good measurements

\[ \Pr(n_{ok} = k) = \binom{m}{k} p^k (1 - p)^{m-k} \]

Probability of having at least $m-q$ good measurements

\[ \Pr(n_{ok} \geq m - q) = \sum_{k=m-q}^{m} \binom{m}{k} p^k (1 - p)^{m-k}. \]
Risk computation using robust set inversion

Computed solution set $\overline{X}$

As long as the assumptions hold, we have $X \subset \overline{X}$.

So

$$n_{ok} \geq m - q \implies x \in X \implies x \in \overline{X}$$

$$\Pr(x \in \overline{X}) \geq \Pr(x \in X) \geq \Pr(n_{ok} \geq m - q)$$

$$r = \Pr(x \notin \overline{X}) \leq \Pr(x \notin X) \leq 1 - \Pr(n_{ok} \geq m - q)$$

$$r \leq 1 - \sum_{k=m-q}^{m} \binom{m}{k} p^k (1 - p)^{m-k}.$$
Bounds determination for a given risk

In practice, we need to solve the inverse problem

The maximum risk "r_{max}" is specified, the bound have to be found

\( \rho \) can be computed by inverting the equation

\[
    r_{max} = 1 - \sum_{k=m-q}^{m} \binom{m}{k} p^k (1 - p)^{m-k}
\]

If the pdf of the noise is known

- Gaussian case

\[
    \alpha = -\Phi^{-1}\left( \frac{1 - \rho}{2} \right)
\]

\[
    [y_{meas}] = [y_{meas} - \alpha \sigma_y, y_{meas} + \alpha \sigma_y]
\]

In practice, GPS provides over-bounding Gaussians
Bounds determination before set inversion

Algorithm input: Chosen risk
- Count the number of available measurements,
- compute a confidence interval for every measurement,
- Use over-bounding Gaussians

\[ [y_{meas}] = [y_{meas} - \alpha \sigma_y, y_{meas} + \alpha \sigma_y] \]

- determine the bounds on every measurement
Bounds can be computed in advance

Example with $r = 10^{-7}$

Risk “1-$p$” for each interval

<table>
<thead>
<tr>
<th></th>
<th>$m = 4$</th>
<th>$m = 5$</th>
<th>$m = 6$</th>
<th>$m = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 0$</td>
<td>$2.5 \cdot 10^{-8}$</td>
<td>$2 \cdot 10^{-8}$</td>
<td>$1.66 \cdot 10^{-8}$</td>
<td>$1.42 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$q = 1$</td>
<td>* $1.29 \cdot 10^{-4}$</td>
<td>$1.0 \cdot 10^{-4}$</td>
<td>$8.16 \cdot 10^{-5}$</td>
<td>$6.90 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$q = 2$</td>
<td>* $2.93 \cdot 10^{-3}$</td>
<td>* $2.16 \cdot 10^{-3}$</td>
<td>$2.71 \cdot 10^{-3}$</td>
<td>$1.42 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

Number of standard deviations for Gaussian pdf

<table>
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<tr>
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<th>$m = 6$</th>
<th>$m = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 0$</td>
<td>5.57</td>
<td>5.61</td>
<td>5.64</td>
<td>5.67</td>
</tr>
<tr>
<td>$q = 1$</td>
<td>3.83</td>
<td>3.89</td>
<td>3.94</td>
<td>3.98</td>
</tr>
<tr>
<td>$q = 2$</td>
<td>2.98</td>
<td>3.07</td>
<td>3.14</td>
<td>3.19</td>
</tr>
</tbody>
</table>
DTM reduces uncertainty introduced by q-relaxation

- non robust GPS only
- robust 1-relaxed GPS only
- robust 1-relaxed GPS + DTM
Validation

Are the computed location domains relevant?
Evaluation methodology

Goal: to test the method with different risk settings

Availability
- bounding box of the sub-paving
- its radius is compared with a 10-meter Alarm Limit (HAL)

Integrity validation of the solution

![Diagram showing integrity OK, integrity unknown, and integrity loss](image)
Experimental vehicle: Carmen

- GPS antenna
- Velodyne
- Stereo Vision System
- Monocular System
- Radar
- Multi-layer Lidar
- WSS + Gyro

CARMEN
Experiment

- 12th arrondissement in Paris
- Septentrio PolaRx, SNR threshold of 35dBAHz
Results with r=0.01%
Results with $r=10\%$
Results with $r=50\%$
### Availability and integrity statistics

<table>
<thead>
<tr>
<th>Integrity risk</th>
<th>r=0.01%</th>
<th>r=10%</th>
<th>r=50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Availability</td>
<td>37%</td>
<td>54%</td>
<td>56%</td>
</tr>
<tr>
<td>Integrity OK</td>
<td>100%</td>
<td>91%</td>
<td>48%</td>
</tr>
<tr>
<td>Integrity unknown</td>
<td>0%</td>
<td>9%</td>
<td>44%</td>
</tr>
<tr>
<td>Integrity lost</td>
<td>0%</td>
<td>0%</td>
<td>8%</td>
</tr>
</tbody>
</table>

![Bar chart showing availability, integrity OK, integrity unknown, and integrity lost for different risk levels](image)

- **Availability**: Shows the percentage of availability for each risk level.
- **Integrity OK**: Represents the percentage of integrity in the 'OK' state.
- **Integrity unknown**: Displays the percentage of integrity in the 'unknown' state.
- **Integrity lost**: Indicates the percentage of integrity that has been lost.

### Graphical Representation

The bar chart below illustrates the distribution of availability and integrity for different risk levels (r=1e-4, r=0.1, r=0.5). The colors used are:
- **Blue**: Availability
- **Green**: Integrity OK
- **Yellow**: Unknown
- **Red**: Integrity lost
Fault detection and identification
Fault detection and identification

Fault detection
-A fault is detected if there is no solution that fulfills all the constraints

Fault exclusion & identification
-Measurements whose constraint is not satisfied by any of the solution boxes are implicitly excluded
-They can easily be identified
Results in Compiègne

robust 1-relaxed GPS + DTM

- Fault = drift of the satellite clock
Real experiment in Paris
Conclusion

Bounded-error solver for tightly coupling GPS pseudo-ranges and 3D map

Navigable maps provide pertinent information to assist localization process

Basic ideas
- We guarantee the computation
- But the computed domains are not guaranteed
- The method can protect against several outliers but the number has to be fixed in advance.

Real experiments to check if a risk computed using over-bounding Gaussians corresponds to reality
- Tests have been done with high risks to be statistically significant
- We have proposed a methodology to deal with ground truth uncertainty
- Results show that the confidence associated with the solution domain is consistent with reality.
Thank you for your attention!
Associated publications

- Fouque, C. and Bonnifait, Ph. “Multi-Hypothesis Map-Matching on 3D Navigable Maps using Raw GPS Measurements”, IEEE ITSC 10