Error minimization in quantum computation

Background

Quantum computing offers the potential to solve problems that are intractable for any classical computer. For this reason, quantum computers are believed to play a central role for future information technology—a fact that has triggered considerable interest in industry and academic research. For instance, large tech companies such as Google, IBM, and Microsoft, as well as international (e.g. EU's Quantum Technology Flagship) and national (e.g. Wallenberg Centre for Quantum Technology) academic networks, spend considerable resources on quantum computation research.

A circuit-based quantum computer performs efficient computation by applying unitary transformations (gates) to quantum bits (two-level quantum systems). These 'qubits' differ from classical bits in that they can be superposed and entangled—features that enable execution of certain algorithms that are intractable for a classical computer [1]. However, present-day quantum computers are highly sensitive to decoherence and noise that destroy the computational advantage. To unlock the full power of quantum computing, gate robustness and error-resilience must be improved so as to reach below the error threshold needed for fault-tolerant quantum computation [2].

Geometric control is a promising tool for implementing robust gates. It is based on geometric (Berry) phases [3] and their non-Abelian generalizations [4]. These phases are only sensitive to the global geometry of quantum state spaces and are therefore conjectured to be resilient to local errors along the evolution of the quantum states. The non-Abelian feature is essential as non-commuting gates are necessary for universal quantum computation [5]. Geometric gates can be realized experimentally in different quantum computation architectures, such as superconducting qubits [6] and NV centers in diamond [7].

Project description

This PhD project focuses on developing numerical optimization techniques to reduce errors of geometric gates by finding optimal evolution paths in quantum state space.

Geometric gates are path-dependent unitary transformations of the form

$$U_\gamma = \mathcal{P} \exp \left(- \oint_\gamma \vec{A} \cdot \mathrm{d}\vec{\lambda} \right), \tag{1}$$

where $\mathcal{P}$ is path ordering and $\vec{A}$ is a matrix-valued vector potential with matrix elements $\vec{A}_{kl} = \langle \phi_k(\vec{\lambda}) | \nabla_{\vec{\lambda}} | \phi_l(\vec{\lambda}) \rangle$, $|\phi_k\rangle$ spanning the computational subspace. Furthermore, γ is a loop in parameter space $\vec{\lambda}$. We seek γ such that

$$U_\gamma = \hat{U}, \tag{2}$$

where $\hat{U}$ represents the desired gate.

Niskanen *et al.* [8] formulated this inverse problem as an optimization task:

$$\min_{\gamma} \|U_\gamma - \hat{U}\|, \tag{3}$$

where $\|\cdot\|$ denotes a suitable operator norm. There are infinitely many γ such that $U_\gamma = \hat{U}$. We propose to impose additional constraints on γ to single out gate implementations that are both realizable and error-minimizing. Specifically, we will investigate the following options:

- Minimizing the arc length of the loop. A short arc length is desirable because it reduces the exposure to errors along the path. One idea is to minimize the arc length by adding it as a regularization term to Eq. (3).
- Improving error resilience by restricting γ to regions less prone to errors.
- Determining quantum codes, such as different forms of decoherence-free subspaces and subsystems [9], that maximize error resilience.

The project will examine both standard qubit architectures and higher-dimensional 'qudit' [10] implementations of quantum computation.

Involved actors and their roles

This project is interdisciplinary in the sense that it combines development of numerical optimization methods with the physics of quantum computation. Martin has experience in optimization problems constrained by differential equations. Erik has made significant contributions to the theory of geometric quantum computation and geometric phases in quantum information. Under the guidance of the supervisors, the PhD student will formulate physical models that can naturally encompass pure geometric phase evolution and design and implement efficient optimization algorithms that can be applied to these models. The PhD student will be employed at the Division of Scientific Computing. Martin will be the main supervisor and Erik will act as assistant supervisor.

Supervisors

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