ELECTROMAGNETISM IN SWEDISH CRYSTALLINE BEDROCK: BETWEEN WAVE AND DIFFUSION

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Abstract: The purpose of this project is to develop novel and computationally efficient algorithms for solving inverse problems for the general Maxwell's equations relevant to geophysics, including both conduction and displacement currents. In these problems, the goal is to estimate medium parameters in the subsurface, modeled by unknown coefficients like the conductivity, by using a collection of sensors, which probe the subsurface with electromagnetic fields.

1. Introduction and motivation. In 1865, Maxwell showed in his seminal paper "A Dynamical Theory of the Electromagnetic field" that light is an electromagnetic wave, by adding displacement currents to Amperes circuital law [12]. Without these displacement currents, the Maxwell equations governing the evolution of electromagnetic fields would be a diffusion equation, just like the heat equation. The general Maxwell equations (in time-invariant, instantaneously reacting, isotropic media) are

$$-\nabla\times\mathcal{H}+\sigma\mathcal{E}+\partial_t\varepsilon\mathcal{E}=-\mathcal{J}^{\text{ext}} \quad \text{and} \quad \nabla\times\mathcal{E}+\partial_t\mu\mathcal{H}=0.$$

They relate spatial and temporal changes of the electrical field $\mathcal{E}$ and magnetic field $\mathcal{H}$ to each other, in media with conductivity σ, dielectric permittivity ε and permeability μ. These medium parameters are typically space dependent. The fields are excited by an external current density $\mathcal{J}^{\text{ext}}$. The conduction current is the blue term $\sigma\mathcal{E}$ and the displacement current is given by the red term $\partial_t\varepsilon\mathcal{E}$ in Amperes law.

In non-conductive (so-called lossless) materials with $\sigma\approx 0$, such as dry air or glass, electromagnetic fields behave like a wave. In conductive materials, such as salt water or clayey soil, the conduction current is much larger than the displacement current at low frequencies and the electromagnetic field can be well approximated by a diffusion equation – the so-called quasi-static assumption: $\sigma\mathcal{E}\gg\partial_t\varepsilon\mathcal{E}$. We refer to the equation with conduction and displacement currents as the general Maxwell equations.

Electromagnetic fields are used in a wide variety of imaging applications. An automotive radar for instance is used to detect other vehicles and estimate their location. In exploration geophysics, multiple electromagnetic prospecting methods are used to determine the conductivity of the subsurface. Examples are controlled-source electromagnetic (CSEM) or radio-magnetotelluric (RMT) methods. All these applications are examples of electromagnetic inverse problems, where one tries to reconstruct the conductivity σ or dielectric permittivity ε of a geological or hydrological target structure from remote measurements of an electromagnetic field $\mathcal{E}$ and $\mathcal{H}$ as illustrated in Figure 1.

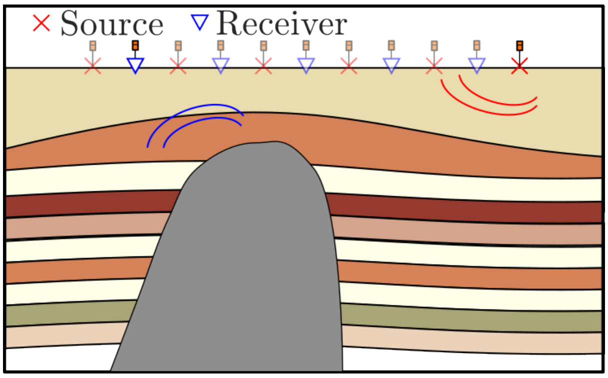


Figure 1: Illustrations of a measurement setup in geophysics. Sources ($\mathcal{J}^{\text{ext}}$) on the earth surface excite electromagnetic fields that penetrate the subsurface. Receivers placed at certain offsets from the source measure the electromagnetic field that interacted with the geological structure. In subsequent experiments source and receiver stations are moved to obtain an illumination of the subsurface with sufficient spatial and temporal coverage. Extracting the conductivity and dielectric permittivity from this data is the goal of the inverse problem.

In the majority of the electromagnetic methods in geoscience, the displacement currents are small and the quasi-static assumption is used in order to form an image of the conductivity of the subsurface. Large parts of Swedish geology, however, contain mildly fractured crystalline bedrock which has a very low conductivity ($\sim 10^{-4}$ S/m or less) and relative dielectric permittivity of 4 to 10, such that the displacement currents and conduction currents are of the same order of magnitude at RMT frequencies of 10 kHz to 1 MHz.

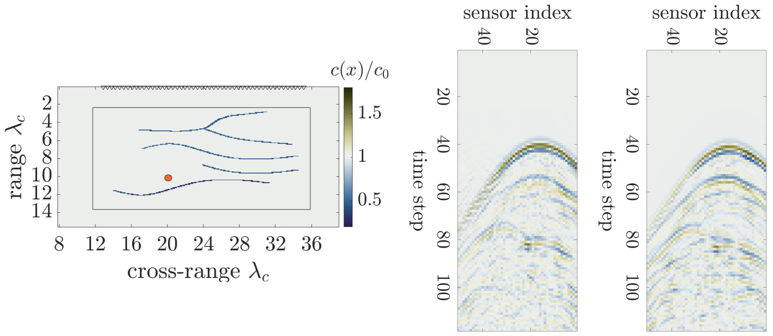
It was shown in [10] that accounting for displacement currents in such geologies leads to better recovery of the subsurface conductivity. In this study, RMT data were inverted for 2D models using regular finite difference meshes without topography and a Gauss-Newton method. However, the dielectric permittivity that relates the electromagnetic field strength to the displacement current was assumed to be constant throughout the subsurface. Despite this limitation, the improvements in data fit compared to a quasi-static inversion scheme were profound. A logical extension to this work is allowing for spatially variable dielectric permittivity and developing mathematical tools for this inverse problem.

In the regime where the electromagnetic fields behave like a wave (low conductivity) and in the quasi-static regime (high conductivity), methods to solve inverse problems are very mature. In this project, we develop novel methods for the electromagnetic inverse problem in the regime where displacement and conduction currents are of the same order of magnitude. This inverse problem is inherently non-linear and ill-posed making it challenging to recover the conductivity and permittivity.

2. Project description. The project seeks to contribute to the practical and theoretical aspects of the electromagnetic inverse problem in media with conduction and displacement currents. To start the project we will revisit the field data case [11] that includes crystalline bedrock studied in [10]. We will invert for a spatially varying permittivity alongside a spatially varying conductivity, extending the EMILIA code in [10, 9, 13]. The sensitivity and correlation of recovered conductivity and permittivity will be studied using an approach based on Fischer information matrices (see [1]). The method of inversion is PDE-constraint optimization where the mismatch between measured and modeled data is minimized in some norm. Next to this we will perform studies on synthetic data in order to establish the impact of the displacement current on the recovered conductivity.

PDE-constraint optimization is a computationally intense iterative method that accounts for the inherent non-linearity of the inverse problem but needs strong regularization that is usually based on ad-hoc priors, and they tend to converge to local minima of the non-convex objective function. Recently, a new class of methods for inverse problems emerged, leveraging ideas from reduced-order modeling. Originally used in control theory as an easy-to-evaluate surrogate model for complex dynamical systems [2], reduced-order models (ROMs) can be used in inversion as an efficient way to encode information about the medium and improve inversion [4, 8, 6].

These ROMs used for inversion are data-driven and are obtained in a nonlinear but computationally tractable algebraic procedure. In the context of this project, ROMs are small dynamical systems that approximate the Maxwell equations and interpolate or fit the measured electromagnetic field responses. They are constructed to respect physics, and to facilitate inversion. They can for instance be used to obtain approximations to so-called internal field, that is, the electromagnetic field inside the unknown subsurface [4, 8, 6]. A good approximation to this internal field linearizes the inverse problem, thereby reducing the computational complexity and making it less likely to converge to a local minimum. An illustration of the approximation of such an internal field is shown in Figure 2 for the case of (lossless) acoustic waves.



Left: Experimental Setup
Center: ROM approximation to the internal field
Right: True internal field

Figure 2: Example of approximating the internal solution of acoustic wave fields using ROMs, as presented in [6]. (Left) In this example, an array of sensors (∇) probes the unknown subsurface with acoustic waves in order to detect fluid-filled fractures. After building a ROM from gathered array data we approximate the wavefield inside the subsurface at the point marked by a red circle. The dimensions are given in terms of the wavelength λ_c, the center wavelength of the used probing pulse in the constant background medium. (Center) ROM approximation to the internal field for every sensor shown in time. (Right) The true internal field shown for comparison. The main scattering events are reproduced in the ROM approximation.

ROM approaches utilizing approximations of the internal field typically minimize some data-misfit, similar to PDE-constraint minimization. However, rather than minimizing the mismatch between measured and modeled data one can also minimize the mismatch between the ROM obtained from the measured data and the ROM obtained from modeled data. This objective function is much better behaved for the inverse problem in the wave-regime [7] at a similar computational complexity as PDE-constraint minimization. So far, almost all ROM-based inversion methods have only been developed for scalar lossless wave equations and for the Maxwell equations in the quasi-static regime [3]. Recently, one such ROM inversion approach was extended to the general Maxwell equations in one-spatial dimension [5].

In the more theoretically oriented part of the project we develop ROM-based inversion approaches for the general Maxwell equations. More specifically, we develop a ROM inversion approach based on approximations of internal fields. This is a non-trivial extension of the approaches in [8, 6] since the construction of the reduced-order model in these works relies on symmetry properties that the lossless Maxwell equations do not have. Such an approach will be compared to the more classical approach in the first part of this project. Thereby we try to address this interdisciplinary problem from multiple angles we extend classical algorithms to invert for spatially varying conductivity *and* permittivity and apply this to Swedish field data. On the other hand, we develop methods in the new class of ROM-based inversion methods in the regime where conduction and displacement current cannot be neglected. Methods in this class are computationally more efficient and less prone to converge to local minima, however, they are not as mature as classical methods in PDE-constraint optimization.

3. Interdisciplinary and international collaboration. This project lies at the intersection of applied geophysics, applied mathematics, and computational science. We will address the electromagnetic inverse problem in the regime of low conductivity for geophysical applications. From the geophysics side, we will study the effect of displacement currents on electromagnetic inversion. One of the goals is to understand under which circumstances a spatially varying permittivity can be recovered from the data and how this impacts the recovery of the conductivity. On the mathematical side, we will develop novel inversion methods based on reduced-order models.

Possible international collaborators for the Ph.D. student on the development of inversion algorithms for the general electromagnetic inverse problem are Liliana Borcea (Department of Mathematics, University of Michigan), an expert on the analysis of electromagnetic inverse problems and Vladimir Druskin (Department of Mathematical Sciences, Worchester Polytechnique institute) an expert in electromagnetic modeling in geophysics.

4. Financial Support. The Ph.D. student will be employed at the Division of Scientific Computing (TDB) at the Department of Information Technology, Uppsala University. The research salary of the Ph.D. student will be financed by CIM (50%) and TDB (50%).

5. Advisors. Jörn Zimmerling is an assistant professor in scientific computing at Uppsala University, specializing in computational inverse problems and numerical methods for partial differential equations. He obtained his PhD from Delft University of Technology in 2018 and spend one semester as a Graduate Student at ICERM at Brown University. He has been a James van Loo Postdoctoral Fellow at the Department of Mathematics at the University of Michigan from 2018 - 2022.

Thomas Kalscheuer has been a senior lecturer in applied geophysics at the Department of Earth Sciences and Uppsala University since 2013 and docent since 2017. Thomas leads the group on electromagnetic geophysics at the Department of Earth Sciences; presently, this group consists of four PhD students, an adjoint professor, and the group leader. Thomas research interests focus on the development of i) multidimensional numerical forward modeling and inversion schemes for data from geophysical electromagnetic (EM) and potential field methods, ii) the joint interpretation of EM, seismic, potential field, and borehole data, and iii) the application to geotechnical, hardrock, tectonic, hydrological, geothermal and volcanological targets.

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