Global Categorical Symmetries and Particle Phenomenology

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Description of the Project

Throughout the history of Physics symmetry principles have always been source of revolutions. The symmetry between observers moving at constant relative velocities with respect to each other led Galileo to propose the principle of relativity, which gave the first insights towards the mathematical foundations of modern physics. It is the symmetry governing the Maxwell equations, the Lorentz group, that led Einstein to generlize Galieo's ideas to the special theory of relativity. From a more modern perspective, the representation theory of the Lorentz group gives the starting point for the theory of relativistic quantum fields. This, together with gauge symmetry, spontaneous symmetry breaking and the Higgs mechanism, is exploited to give the Standard Model of elementary particles, one of the biggest scientific achievements of the 20th century.

There are however several known aspects of elementary particles that go beyond the Standard Model, among the most interesting being the small masses of the neutrinos. Neutrinos are intriguing particles, whose existence was proposed by Wolfgang Pauli to explain the strange trajectories observed in β-decays. Originally, neutrinos were believed to be massless, and the Standard Model (SM) describe them as such. However, a discrepancy with the SM prediction about the structure of the fluxes of solar neturinos indicates that there are neutrino oscillations. The latter can be explained by very small mass terms between various flavors of neutrinos. The neutrino masses are incredibly small[1], suggesting that their masses are protected by a symmetry [2].

Recent research in quantum field theory is undergoing a further revolution with the discovery of various novel kinds of symmetries associated to extended operators. These generalized global symmetries [3] include higher-form symmetries, categorical symmetries such as higher-group symmetries or non-invertible symmetries, or even more generally sub-system symmetries, *etc.* Radically extending the standard notion of symmetry based on the mathematics of Lie algebras and Lie groups, these novel symmetries are based on advanced mathematical structures generalizing higher-groups and higher-categories. Generalized symmetries are expected to have profound implications for our understanding of the dynamics of quantum fields relevant in various areas of physics ranging from condensed matter physics to quantum information, high-energy physics, and even cosmology.

In this interdisciplinary project we propose exploring the implications of the higher categorical structure of chiral symmetry [4] for physics beyond the Standard Model [5]. The mathematical structures governing the generalized global symmetries relevant for this project are the so-called higher n-fusion categories, which are very interesting mathematical objects in their own right [6]. In particular, the project will explore the implication of higher categorical structure for naturalness constraints on neutrino masses following a recent proposal by Córdova and Ohmori [7].

[1] The current bound on neutrino masses set them to be lower than 0.2 eV as opposed to the 0.511 MeV of the electron [1].

Interdisciplinary Aspects of the Project

This project fulfils the main commitment of CIM to support collaboration between researchers in mathematical topics and other academic fields, here we highlight the relevant mathematical aspects of the proposal as well as the physical applications.

Mathematical Aspects of the Proposal. The mathematical structures governing generalized global symmetry and their representations are currently being developed by the community of mathematicians working in higher category theory and representation theory. The starting point for such theory is the well-developed theory governing generalized symmetries of two dimensional theories in terms of modular tensor categories and quantum groups. The generalization of this structure to higher dimensions is provided by n-fusion categories, whose foundations are currently being developed by the mathematicians Johnson-Freyd and Reutter, building on the well-known results by Freed, Hopkins and Teleman in the context of the classification of topological orders and its interplay with Lurie's ideas on the cobordism hypothesis. At the mathematical level, this project will dwell in the structure of 3-fusion categories, which are the ones relevant for the generalised symmetries of four-dimensional field theories. The latter have en extremely rich structure which includes various types of higher groups as examples. In particular, we are after two main classes of mathematical results: 1) structural theorems about 3-fusion categories, and 2) an obstruction theory extending to these structures the more familiar theory of characteristic classes. The main examples we will use in this PhD project are the ones which are relevant for physics applications, namely the 3-fusion category associated to chiral symmetries for theories with a non-trivial ABJ anomaly in four dimensions.

Physical Aspects of the Proposal. An important open problem in the context of the physics beyond the standard model is the naturalness problem. Of pivotal interest for this project are the neutrino masses, which are surprisingly small but non-zero, thus indicating new physics beyond the standard model. The fact that neutrino masses are so small is an example of a naturalness problem. A possible solution to such a problem is that typically, if parameters are small, it is because they are protected by a symmetry which is softly broken. Recently, Córdova and Ohmori proposed that a categorical symmetry which generalises the well-known chiral symmetry to theories with an ABJ anomaly can serve the purpose of constraining neutrino masses [7]. This is a new and very interesting application of generalised symmetries that needs to be explored in the context of beyond the Standard model physics. In order to fully develop this concept, more insights on the mathematical structure of chiral symmetry are necessary, which gives an intrinsic interdisciplinary character to this project.

Personel and their Roles

PhD Candidate. This project is intrinsically interdisciplinary: it has a mathematical core and several physical applications. The relevant mathematical structure for this project are 3-fusion categories. A close interplay with the representation theory group at the mathematics department is expected here. In particular, we expect a direct interplay with works by Kragh and Mazorcuk. The main physical applications of the project are the phenomenological constraints that emerge from the categorical structure of chiral symmetry on the physics beyond the Standard model. In particular, we will study the new naturalness constraints that arise from generalized global symmetries. The ideal PhD candidate for this project is a student that has a stellar background in quantum field theory and a strong drive towards understanding its mathematical aspects.

Main advisor. The main advisor for this PhD project is Dr. Michele Del Zotto, senior lecturer at the Geometry and Physics group within the Department of Mathematics of Uppsala University. He is a renowned expert in the mathematical aspects of quantum field theory with a proven track record of successful supervision. Among the results of Dr. Del Zotto are several related to the main aim of this project. This includes the generalization of the 't Hooft screening argument to solitonic defects of higher dimensions, that leads to the notion of defect groups [8], and the explicit construction of infinitely many new examples of categorical symmetries in four-dimensional quantum field theories [9]. Recently, Dr. Del Zotto became deputy director of the Simons Collaboration in Global Categorical Symmetries (SCGCS) and in that role he is working on developing the mathematical structure of global categorical symmetries of quantum fields. The latter is a collaboration at the interface of mathematics and physics founded by the prestigious Simons Foundation (NY), which includes some of the most important groups worldwide in the field, including the mathematics departments of Harvard Univeristy, the University of California at Berkeley, the BIMSA group at Tsinghua University, as well as the IHES in Paris.

First co-advisor. The first co-supervisor of this PhD project is Prof. Joseph Minahan from the Theoretical Physics Division at the Department of Physics and Astronomy of Uppsala University. He has been mostly active in the context of holographic duality and strongly coupled quantum field theories, which are topics in close contact with the main line of research in this project. Prof. Minahan has a successful track record of grant applications and fundings and he is presently the Editor-in-Chief for Journal of Physics A, the top journal for theoretical physics in the United Kingdom. His expertise in the theory of quantum fields will be a great asset for this project.

Second co-advisor. The second co-supervisor for the project is Clay Córdova, whom is a Neubauer Family Assistant Professor at the University of Chicago. He was a long-term member at the Institute for Advanced Study, and a Junior Fellow at the Harvard Society of Fellows. He completed his PhD in Physics at Harvard University. Córdova has broad interests in quantum field theory with particular emphasis on the ideas of symmetry, duality, and their implications for renormalization group flows. He is also keenly interested in the interplay of new mathematical tools with these subjects. Some of his key results related to the proposed project are developing new notions of higher group symmetry in quantum field theory, understanding the interplay between anomalies and the mass gap in field theory as well as the first applications of categorical symmetry to constrain beyond the SM physics, which we plan to explore systematically. Thanks to the support of the SCGCS, the Córdova group in Chicago will host research visits by the selected PhD candiate during the later stages of the project. This will enhance the opportunities of collaboration for the candidate and assist their scientific career in this exciting and rapidly developing field.

Cofinancing

50% research salary co-financing of the selected PhD candidate will be provided by the SCGCS Simons Foundation grant. The selected PhD candidate will become part of the group of PhD students of the SCGCS. The SCGCS will organize several schools and conferences during this PhD project and the close contact with other researchers will give the selected PhD candidate many opportunities for growth, both at the scientific level as well as establishing connections for future collaborations. We expect that this will help attract top-class candidates to this position.

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