Machine Learning the Geometry of String Theory

Goal

String theory promises to give a complete description of the physics of the universe. A crucial step towards this goal is to construct and analyse different string compactifications. There are many fundamental open questions about the geometry underlying these constructions. The main goal of this project is to attack some of these questions using tools from machine learning. In particular, machine learning will be used to construct string compactifications, and chart the mathematical structures underlying these constructions. Conversely, string compactifications will be used as a testing ground for machine learning.

Background

String theory is a mathematically rigorous framework that unifies the cornerstones of modern physics: quantum field theory and general relativity. As such, string theory promises to give a complete description of the physics of the universe, ranging from microscopic to macroscopic scales. However, string theories live in ten dimensions. In so-called string compactifications, the extra dimensions curl up and remain hidden at low energy scales. A side effect of this process is that the geometry and topology of the hidden, compact dimensions determine the laws of physics for low-energy observers. Consequently, physical problems translate into geometry puzzles. At the same time, via string compactifications the tools of physics may be used to study geometry, topology, and related areas of mathematics.

String compactifications may be constructed in a multitude of ways, leading to a vast landscape of physical theories that all abide the stringent constraints of quantum gravity. However, identifying, within this landscape, a theory fully describing our Universe is a great challenge. In the past 5 years, machine learning (ML) has emerged as a versatile tool for this quest. ML provides efficient methods to explore and exploit string compactifications, and identify configurations matching observations. Moreover, ML gives new insights into geometry and topology, and allows us to compute features that are inaccessible with analytic tools. This has revealed tantalizing hints of new mathematical structures. In this project, we will explore one or several of these promising research directions, depending on the profile of the successful applicant.

Project Plan

A first aim of this project is to make use of the novel developments at the intersection of machine learning and string compactifications to construct more realistic physical theories. The main focus will be on 6-dimensional Calabi-Yau (CY) spaces, which satisfy necessary string theory constraints from the get-go. CY spaces contain rich geometric structures, both from the point of view of algebraic geometry, Riemannian geometry, and also symplectic geometry. All these structures interplay in an interesting way, which provides a mathematical motivation to study CY geometry. Moreover, CY spaces can be constructed in great numbers using algebraic geometry. Algebraic geometry also provides methods to compute certain topological features of CY spaces. This is important for string compactifications, since it allows us to determine e.g. the spectrum of fundamental particles of the resulting physical theory. In the past, this has been used to construct a large number of semi-realistic particle physics models (Ref1) and, more recently, ML techniques have been used to overcome hurdles related to computational complexity in this field (Ref2).

Despite this success, there is one important feature of CY geometry that cannot be computed analytically: the unique, Ricci-flat CY metric which determines e.g. interaction strengths in the theories that result from string compactification. Lacking analytic tools, one may compute the metric numerically. ML techniques allows us to not only compute the metric, but do this efficiently. Thus, the learned metric can be used in physics computations, and this will be the first aim of this project. Open source ML packages for CY metrics, e.g. Ref3 supported by the main supervisor's group and Ref4 by other researchers, will be developed further to address open physical problems such as Yukawa coupling normalisations, supersymmetry breaking and inflation in string theory.

A second branch of this project is to study the ML results for CY spaces with the aim of providing new insights in mathematics. String theory research of CY geometry has a history of revealing mathematical structures, like the first hints of mirror symmetry between pairs of CY spaces (Ref5). Machine-learned CY metrics may help uncover new structures. For example, they may be used to test aspects of mirror symmetry, such as the SYZ conjecture (Ref6), or predictions for relations between algebraic and symplectic invariants (Ref7). Central to these are Lagrangian submanifolds, which are subspaces of CY spaces that interplay in a certain way with the CY metric. We currently have a poor understanding about what governs the existence of the Lagrangians. ML has potential to give a truly novel approach to this question.

ML also allows us to bypass the (often computationally expensive) algorithms (Ref8) that are used to compute topological quantities for CY spaces (Ref9, Ref10). Recently, this has given empirical evidence that analytical formulae govern certain topological aspects related to CY spaces (Ref10). This novel structure has yet to find its mathematical explanation. Furthermore, there is scope to extend ML studies of geometry and topology beyond the CY regime. Applying ML tools in such settings would be novel, and may inspire new mathematical developments.

This project falls under the umbrella of physics-informed ML where knowledge of physical laws is used in addition to data to train ML models. Here, a popular approach is to use deep learning and directly modify the neural network architectures with physics knowledge, for instance, so that the solutions satisfy certain boundary conditions, see Ref11 for several examples. Another popular approach is to modify the loss function so that physics constraints are added as soft penalty constraints instead of solely using data that fits a set of input-output pairs. The success of this approach for computing metrics of string compactification spaces has been illustrated by our recent work in Ref3. In this project, we will advance these lines of work. Connections between soft penalty constraints, Bayesian formulations and uncertainty quantification; as well as connections between scalable multi-task learning in ML and enforcing several physics constraints are expected to be promising starting points.

Finally, this project provides an opportunity to use string compactifications as a setting for experiments in ML: geometries are abundant, large data sets can be generated at relative ease, and well-understood physics and mathematics constraints can be used to inform neural networks. This will allow to explore e.g. the scaling behaviour of machine learning in a controlled physics-informed setting, and interpretability of ML models.

Interdisciplinary Aspects

The success of this project relies on combining methods from theoretical physics, mathematics, and machine learning. To ascertain that the PhD student will be trained in these different fields, she/he will be supervised through a new collaboration including experts in these fields. The main supervisor, Magdalena Larfors, at the Theoretical Physics group, is a string theorist who studies string compactifications using tools from geometry in conjunction with ML techniques. The co-supervisor Ayca Özcelikkale is an ML expert in the Signals and Systems division working on development of application specific ML approaches in various fields, such as factory logistics and communication networks, as well as development of neuroscience inspired ML approaches. The second co-supervisor Georgios Dimitroglou Rizell, in the Geometry and Physics group at the Department of Mathematics, is a symplectic geometer with exhaustive expertise in the geometry and topology of CY spaces. The cross-disciplinary environment of CIM will also be of great value for the PhD student, as it provides a network where all technical aspects of the project can be discussed.

Main Supervisor

Magdalena Larfors, Department of Physics and Astronomy, Division of Theoretical Physics, magdalena.larfors@physics.uu.se

Co-Supervisors

Ayca Özcelikkale, Department of Electrical Engineering, Signals and Systems, ayca.ozcelikkale@angstrom.uu.se

Georgios Dimitroglou Rizell, Department of Mathematics, Geometry and Physics group, georgios.dimitroglou@math.uu.se

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