Generalized Symmetries and Phases of Quantum Fields

HOST INSTITUTION: Uppsala University

ASSOCIATE: Michele Del Zotto
Senior Lecturer, Department of Mathematics
Uppsala University

CO-ASSOCIATE: Joseph Minahan
Professor, Department of Physics and Astronomy
Uppsala University

Description of the Project

Symmetry is a type of invariance: the property that a mathematical object remains unchanged under a set of operations or transformations. Symmetry transformations of physical systems are among the building blocks for the understanding of the physical laws of Nature. The symmetry between observers moving at constant relative velocities with respect to each other led Galileo to propose the principle of relativity, which gave the first insights towards the foundations of modern physics. It is the symmetry governing the Maxwell equations, the Lorentz group, that led Einstein to generalize Galileo’s ideas to the special theory of relativity, which is at the foundation of our understanding of the kinematics of elementary particles as well as the stability of nuclei. In the quantum realm, thanks to the deep interconnections between spin and statistics, one can explain the periodic table of elements starting from symmetry. From a more modern perspective, the representation theory of the Lorentz group gives the starting point to start organizing the theory of relativistic quantum fields. Quantum numbers of elementary particles are organized by symmetry groups. This, together with gauge symmetry, spontaneous symmetry breaking and the Higgs mechanism, is exploited to formulate the Standard Model of elementary particles, one of the biggest scientific achievements of the 20th century.

Recent research in quantum field theory is undergoing a further revolution with the discovery of various novel kinds of symmetries associated to extended operators. These generalized global symmetries include higher-form symmetries, categorical symmetries such as higher-group symmetries or non-invertible symmetries, or even more generally sub-system symmetries, etc. Radically extending the standard notion of symmetry that was previously based simply on the mathematics of Lie algebras and Lie groups, these novel symmetries are based on more advanced mathematical structures generalizing higher-groups and higher-categories. Generalized symmetries are expected to have profound implications for our understanding of the dynamics of quantum fields relevant in various areas of physics ranging from condensed matter physics to quantum information, high-energy physics, and even cosmology.

In this interdisciplinary project we propose exploring the implications of the higher categorical structure of symmetry towards applications in the classification of phases of matter. The mathematical structures governing the generalized global symmetries relevant for this project, which are very interesting mathematical objects in their own right. In particular, the project will explore the implication of higher categorical structure for the structure of infrared phases of matter, vastly generalizing the Landau Paradigm for the classification of phases in terms of patterns of symmetry breaking.

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1 For some recent applications to the physics of the standard model see eg. 2, 3.
Interdisciplinary Aspects of the Project

This project fulfils the main commitment of CIM to support collaboration between researchers in mathematical topics and other academic fields, here we highlight the relevant mathematical aspects of the proposal as well as the physical applications.

Physical Aspects of the Proposal. An important open problem in the context of the physics of quantum field theories is that of the classification of infrared phases. A well-known proposal by Landau is that phases can be classified according to patterns of symmetry breaking, in terms of order parameters. It is well-known that exploiting point-like order parameters the Landau paradigm has several exceptions, perhaps the most prominent is given by the Berezinskii–Kosterlitz–Thouless transitions. Exploiting extended operators as order parameters a higher Landau paradigm might be accessible where a finer structure of phases can be decoded from patterns of higher symmetry breaking. The implications of this idea are focus of current research – see eg. [5] for a very recent paper on the topic.

Mathematical Aspects of the Proposal. The mathematical structures governing generalized global symmetry and their representations are currently being developed by the community of mathematicians working in higher category theory and representation theory. The starting point for such theory is the well-developed theory governing generalized symmetries of two dimensional theories in terms of modular tensor categories and quantum groups. The generalization of this structure to higher dimensions is provided by $n$-fusion categories, whose foundations are currently being developed by the mathematicians Johnson-Freyd and Reutter, building on the well-known results by Freed, Hopkins and Teleman in the context of the classification of topological orders and its interplay with Lurie’s ideas on the cobordism hypothesis. At the mathematical level, this project will dwell in the structure of 3-fusion categories, which are the ones relevant for the generalised symmetries of four-dimensional field theories. The latter have an extremely rich structure which includes various types of higher groups as examples. In particular, we are after two main classes of mathematical results: 1) structural theorems about 3-fusion categories, and 2) an obstruction theory extending to these structures the more familiar theory of characteristic classes. The main examples we will use in this PhD project are the ones which are relevant for physics applications, namely the 3-fusion category associated to chiral symmetries for theories with a non-trivial ABJ anomaly in four dimensions.

Personel and their Roles

PhD Candidate. This project is intrinsically interdisciplinary: it has a mathematical core and several physical applications. The relevant mathematical structure for this project are higher fusion categories. A close interplay with the representation theory group at the mathematics department is expected here. In particular, we expect a direct interplay with works by Kragh and Mazorcuk. The main physical applications of the project are the constraints that emerge from the categorical structure of symmetry on the physics of infrared phases of quantum fields. The ideal PhD candidate for this project is a student that has a stellar background in quantum field theory and a strong drive towards understanding its mathematical aspects.

Main advisor. The main advisor for this PhD project is Dr. Michele Del Zotto, senior lecturer at the Geometry and Physics group within the Department of Mathematics of Uppsala University. He is a renowned expert in the mathematical aspects of quantum field theory with a proven track record of successful supervision. Among the results of Dr. Del Zotto are several related to the main aim of this project. This includes the generalization of the ’t Hooft screening argument to solitonic defects of higher dimensions, that leads to the notion
of defect groups [6], and the explicit construction of infinitely many new examples of categorical symmetries in four-dimensional quantum field theories [7]. Of particular interest are developments associated to the higher structure of symmetry, for a first study in the context of quantum electrodynamics see [8]. Dr. Del Zotto is the deputy director of the Simons Collaboration in Global Categorical Symmetries (SCGCS) which is a further asset to this project, offering opportunities of networking and collaboration to the successful candidate.

Co-advisor. The proposed co-supervisor for this PhD project is Prof. Joseph Minahan from the Theoretical Physics Division at the Department of Physics and Astronomy of Uppsala University. Prof. Minahan has been mostly active in the context of holographic duality and strongly coupled quantum field theories, which are topics in close contact with the main line of research in this project. Prof. Minahan has a successful track record of grant applications and fundings and he is presently the Editor-in-Chief for Journal of Physics A, the top journal for theoretical physics in the United Kingdom. His very broad expertise in the theory of quantum fields will be a great asset for this project.

Cofinancing

50% research salary co-financing of the selected PhD candidate will be provided by the Department of Mathematics. Moreover, the PhD student will be in close contact with the activities organized in the context of the VR Excellence Centre for Geometry and Physics of Uppsala University and the Simons Collaboration for Global Categorical Symmetries (SCGCS). The SCGCS will organize several schools and conferences during this PhD project and the close contact with other researchers will give the selected PhD candidate many opportunities for growth, both at the scientific level as well as establishing connections for future collaborations. We expect that this will help attract top-class candidates to this position.

References