Large Scale Combinatorial Optimization for Phase Correction in Magnetic Resonance Imaging

Background and Aims

Magnetic Resonance Imaging (MRI) depicts the magnetization of the nuclear spins in the object under study. Since the MR signal reflects the magnetization component transverse to the magnetic field at the time of measurement, MR images are fundamentally complex-valued. In most scenarios, only the signal magnitude is of significance, but in several important clinical applications the image phase is used to store information of interest (1, 2, 3). In these applications, interpretation problems emerge linked to the periodic nature of the image phase, bringing a need to correct or unwrap the phase. Due to the challenges involved in phase unwrapping, we believe that applications utilizing phase information are underutilized in clinical practice.

The overall aim of this project is to solve the problem of phase unwrapping or phase correction in various instances of MRI. To do this, we propose to formulate combinatorial optimization problems over the whole set of pixels. For phase unwrapping, the decision variables are one integer at every pixel, corresponding to the “period number” or “wrap count”. The configuration is then sought that minimizes the spatial smoothness of the phase. Solving such optimization problems is challenging, and the problem of finding a global minimizer is usually NP-hard. Similar problems, however, arise in many areas of computer vision such as image restoration, segmentation and stereo vision. Therefore, a wide array of optimization algorithms already exists for this class of problems. Methods of special interest include message passing algorithms (4) and graph cuts (5). Since the number of decision variables is on the order of millions, efficient optimization methods are required.

More precisely, we will consider three specific MRI applications, described in the “Applications” section below. These applications have in common that reliable phase correction is required for clinical utility. We aim to develop methods which achieve as good phase unwrapping performance as possible for these applications within clinically acceptable processing time. This includes proposing mathematical representations of these imaging scenarios, formulating corresponding combinatorial optimization problems, and identifying suitable existing optimization algorithms and possibly developing novel ones where needed. The algorithms will be tailored to fit the problems at hand, evaluated with respect to algorithm parameters, and compared to each other in terms of quantitative and qualitative measures. Evaluation will be performed on MRI data acquired from clinical patients.

The success of this project depends on interdisciplinary collaboration, requiring expertise in both MRI physics and combinatorial optimization. Our planned project team combines competence in these different research areas.

Applications

Unwrapping of Phase Contrast MRI

The image phase can be made sensitive to motion through the use of velocity sensitizing magnetic gradient fields (6). In phase contrast MRI, the difference is taken between two interleaved acquisitions with varying amounts of velocity encoding to cancel out confounding effects. The phase of the difference image then becomes proportional to the spin velocity only, enabling the quantification of flow through the major blood vessels. The velocity-to-noise ratio is inversely proportional to the amount of velocity encoding ($v_{enc}$), but if $v_{enc}$ is too low, phase wrapping occurs (7). Flow quantification errors due to such velocity aliasing can be repaired by phase unwrapping. The image acquisition is ECG-triggered to allow temporal resolution across the cardiac cycle. The resulting images therefore have a cyclic time dimension in addition to two or three spatial dimensions. Existing unwrapping methods for this application are typically based on path-following algorithms (8, 9) or approximate inversion of the Laplacian (10, 11). We hypothesize that combinatorial optimization approaches such as TRW-S (12) or Iterative Graph Cuts (13) will achieve more accurate unwrapping, which is supported by our preliminary results (Fig. 1).

Referenceless PSIR for LGE

Late gadolinium enhanced imaging (LGE) is a basic component of a cardiac MR examination (14). It is acquired about ten minutes after administration of gadolinium contrast agent to identify non-viable tissue in the myocardium. Phase Sensitive Inversion Recovery (PSIR) increases the image contrast by exploiting the full dynamic range from negative to positive polarity, but requires demodulation of interfering background phase which is usually estimated from a
Fat/Water Separation

In the clinical setting, the MR signal derives from hydrogen nuclei in water and fat molecules. The resonant frequency of the magnetization differs slightly between water and fat, which enables encoding information about the water/fat content into the image phase and separating the two components during post-processing (3). This separation is confounded by the background phase which introduces an ambiguity between water- or fat dominance, which can be resolved under the assumption of spatial smoothness of the background phase. Phase correction methods exist which are based on path-following algorithms (17, 18) or graph cuts (19). The best-performing current method only requires a single QPBO graph cut (20, 21). However, since the problem is not submodular, the cut may leave unlabeled pixels which can be determined using some heuristic (22). Our initial experiments indicate that high quality phase correction can be achieved using binary graph cuts which are globally optimal with respect to the max-norm (23), using our recently developed algorithm (24) and avoiding the use of ad-hoc heuristics.

Implementation and Evaluation

Image data collection from patients is ongoing at Uppsala University Hospital under permission from the Swedish Ethical Review Authority (diary number 2022-03289-01). Unwrapping of phase-contrast MRI will be evaluated on synthetically wrapped high-venc data using the acquired data as ground truth. Phase correction of referenceless PSIR will be compared quantitatively and qualitatively with PSIR corrected using a reference image. The developed fat/water separation algorithms will be evaluated on a benchmark dataset including ground truth (25).

Research Environment

The Ph.D. student will be employed in the Molecular Imaging and Medical Physics research group at the Department of Surgical Sciences with a primary workplace at Uppsala University Hospital. The salary will be co-financed by the Centre for Interdisciplinary Mathematics (50%) and ALF funds at Uppsala University Hospital (50%).

Main supervisor: Johan Berglund is an Associate Professor at the Department of Surgical Sciences, Uppsala University. His research concerns method development of MRI in the clinical setting with special focus on motion correction, fat/water separation and cardiac MRI. He is head of the MR physics group at Uppsala University Hospital. johan.berglund@akademiska.se

Co-supervisor: Filip Malmberg is an Associate Professor at the Department of Information Technology, Uppsala University. He is a researcher within computerized image analysis, with a focus on combinatorial optimization methods. filip.malmberg@it.uu.se

Ph.D. student: The ideal candidate should have strong programming skills and a solid mathematical background with an interest in mathematical sciences, signal processing, computer vision, and medical applications.

The work will be carried out in parallel with the project Lexicographic Max-Ordering Optimization For Data Analysis On Graphs (PI Malmberg) starting 2024, funded by the Swedish research council, with the aim to develop novel methods and algorithms for solving combinatorial optimization problems similar to those encountered in the present project. We therefore expect strong synergies between the projects, and envision a close collaboration between the Ph.D. students working in the two projects.
References


