Motivation

Quantum field theory provides a quantitative explanation of natural phenomena as diverse as the fluctuations in the cosmic microwave background, superconductivity, and elementary particle interactions in colliders. Its success is evinced by an excellent agreement between its predictions and experiments. Quantum field theory is also an essential part of the effort to study quantum theories of gravity, for instance it allows us to define non perturbatively some quantum gravitational systems via holography and the AdS/CFT correspondence. Finally many concepts in advanced mathematics (e.g. in algebraic topology) find a use in quantum field theory; vice-versa the study of quantum field theory can lead to new ideas in pure mathematics.

The structure and dynamics of quantum field theories, however, are still poorly understood away from the perturbative regime. Many natural phenomena are characterized by strong self-interactions, and their analysis requires going beyond perturbation theory techniques. The project aims to develop exact methods to study the dynamics of strongly coupled field theories.

One of the over-arching themes of the project is the use of symmetries to study the dynamics of quantum field theory, in particular supersymmetric field theories. Supersymmetry gives us powerful methods to study strongly coupled systems that display phenomena of phenomenological relevance, like confinement or the breaking of chiral symmetries. It is also one of the most important candidate mechanisms of stabilization of the weak scale, and the search for its signatures is part of intense experimental efforts at colliders like LHC. In condensed matter supersymmetric systems can arise, for instance, on the boundary of topological insulators.

General Idea and Aims

Important information about the dynamics of a physical system can be obtained by studying its response to external applied fields (e.g. by placing it in a magnetic field). The geometry of space-time itself can be considered as such an external field. It is possible to derive a multitude of exact results in supersymmetric quantum field theories by exploiting the possibility to place them on curved manifolds preserving some supersymmetry. Certain quantities of physical interest can then be computed exactly. Notable results have arisen from considering specific manifolds (e.g. spheres) and have allowed the definition of very precise nonperturbative probes of the dynamics of these systems [1-2]. The aim of the doctoral thesis is to go beyond case-by-case studies and make full use of supersymmetry in curved space, reaching interesting new results about the dynamics of strongly coupled QFT's.

A powerful tool to study in generality rigid supersymmetric theories on curved manifolds $\mathcal{M}$ is to regard the geometry of space as a background (super)gravity field [3]. Taking advantage of this framework one can show that there are classes of theories that constitute a middle ground between generic QFTs, with their very rich dynamics, and Topological Field Theories, with their deep connections to Mathematics [4]. This is a promising, largely unexplored terrain both in theoretical
physics and mathematics. The aims of the project will be to:

- Classify the observable quantities in these theories and study methods to determine them exactly even at strong coupling by making use of symmetry.
- Use these new probes to study the dynamics of strongly interacting quantum systems.
- Study how physical observables in these theories depend on the geometry of space-time and use this information to relate quantum field theories in different dimensions (e.g. by taking the limit in which one of the dimensions of spacetime becomes very small)
- Understand and develop the proper mathematical framework to describe these extensions of topological field theory.
- Find out what is the mathematical interpretation of the supersymmetric observables: what information do they convey about the geometry of the manifold on which the theory lives on?

**Interdisciplinary aspects:**

There are several aspects of the project lying at the interface between physics and other fields with connections to mathematics:

- Equivariant localization, a mathematical technique, which has its roots in symplectic geometry, by which the path integral of certain supersymmetric field theories can be reduced to a finite dimensional integral. Indeed using supersymmetry it can be shown that the path integral does not depend on certain parameters. As these parameters are made large the support of the path integral becomes finite dimensional. In this project we will study how localization is implemented in a large new class of field theories.

![Fig 1: Simple example of localization. As the deformation parameter t is varied the area below the curves is constant but localizes at a point with spread t^{-\frac{1}{2}}.](image)

- Superconformal field theories include protected sectors with a very rich and interesting structure. For instance protected operators in N=2 SCFT in four dimensions have correlators displaying an infinite chiral symmetry [5]. This links to pure mathematics research into Vertex Operator Algebras and Integrable systems. These sectors have been studied so far mainly in flat space. We will develop a framework to study their properties in curved space.

- Some of the recent investigations into Supersymmetry in curved space have been made possible by using machine computing. For instance it allows to construct high dimensional couplings of supersymmetric field theories to supergravity. One of the aims of the project is that of systematizing the use of computers and providing the community with automation tools that are widely applicable.

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Advisors:

Guido Festuccia's main interest is the study of quantum field theory and their applications. He has developed background supergravity methods to study supersymmetric field theories in curved space. Jian Qiu is working at the interface of geometry and mathematical physics (e.g. applications of toric geometry). He is an expert in the use of localization techniques in quantum field theory. Michele del Zotto research includes the study of superconformal theories in diverse dimensions and their relation with string/M theory.

References