Spreading phenomena, resilience and modularity in probabilistic graphs applied to disaster science

Random graphs are probability distributions over graphs that can be used to study the properties of families of graphs. The best known random graph models, $G(n,p)$ and $G(n,m)$, use a single parameter ($p$ or $m$) to describe the probability of adjacency of any pair of vertices; while this makes the analysis of several graph properties straightforward, the families of graphs represented by these models are different from most graphs representing real systems, such as online information networks and road networks. Probabilistic graphs, $G(V,E,P)$, are more general models where a function $P : V \times V \rightarrow (0,1]$ allows a fine-grained control over the probabilities associated to specific pairs of vertices. Constraints on the function $P$ can be used to define specific models, for example, a constant function $P$ would correspond to a $G(n,p)$ random graph.

The objective of this project is to define families of probabilistic graph models that can be used to represent real interconnected systems that are relevant in the field of disaster science and study their properties. Disaster scientists in many context use graphs as mathematical representations. However, the availability of information in the context of disasters is often limited, leading to uncertainty about the existence of edges and vertices in the graphs.

In particular, this PhD project will focus on three properties. The first concerns spreading processes. This part of the project will be based on well studied epidemic models, such as SIS and SIR, under the presence of uncertainty in the existence of the edges. The second property, very related to the first, is resilience, that is, the ability of (a significant portion of) the graph to remain connected when nodes or edges are removed from it. This will be based on the study of percolation processes. The third property is modularity, that is, the extent to which the graph contains subgraphs having a high density and a limited connectivity to vertices outside the subgraph.

**Interdisciplinarity:** This work is in collaboration with the Centre of Natural Hazards and Disaster Science ([CNDS](#)). The families of graphs studied in this project will be defined to model interconnected systems that typically emerge during or after natural disasters. Two relevant examples are the networks of individuals and authorities sharing information, e.g., about areas that have been damaged, areas to avoid, general instructions or people who have been marked as safe. The vertices of these networks can be actors (citizens and institutions), or they can represent the content of the conversation, for example a network of word co-occurrence can be used to summarize
the main topics under discussion during and after the event. A second example are networks of critical infrastructure, where the probability that infrastructure becomes unusable may depend on the geological properties of the area.

Random graphs are often used as null models to define and test hypotheses about the properties of real data. The experimental part of the project will be based on real data, for example social media communication networks during a natural disaster, and the mathematical models developed by the student will be used to study the properties of these real networks.

**Content of research in a mathematical field and in an applied field:**
The study of random graphs is a main sub-field of mathematics. The focus of this project is on complexity science and network science, where probabilistic graphs are used as tools to analyze real data.

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