Interval Computations in Metrology

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1. Need for Data Processing

- In many engineering situations, we need to make decisions.
- Some of these decisions are made by humans, some by automatic control systems.
- The decisions y are based on the valued of the relevant quantities $x_1, \ldots, x_n$: $y = f(x_1, \ldots, x_n)$.
- Ideally, the values x_i should come from measurement.
- However, in many cases, we also need to use expert estimates.
- This is typical, e.g., in inverse problems, which are, in general, ill-defined.

2. Need for Expert Estimates

- For example, we may be interested in the value $x(t)$, but sensors only measure averages

$$x_{\rm av}(t) = \int_{t-\varepsilon}^{t+\varepsilon} K(t-t')\cdot x(t')\,dt \text{ and } \int K(\tau)\,d\tau = 1.$$

- To make these problems well-defined, we need to add prior information – which comes from experts.
- For example, in measuring $x(t)$, the experts can give us the upper bound M on the rate of change $|\dot x(t)|$.
- In this case, $|x(t) - x_{\rm av}(t)| \le M\cdot\varepsilon$.
- Both measurement results and expert estimates come with uncertainty.

3. Need to Take Uncertainty into Account

- Measurements are never absolutely accurate.
- The measurement result $\widetilde{x}$ is, in general, different from the actual value x of the corresponding quantity.
- Ideally, we should know the probability distribution for the measurement error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$.
- However, in most practical cases, all we know is the upper bound Δ on the measurement error: $|\Delta x| \le \Delta$.
- In this case, once we have a measurement result $\widetilde{x}$, all we know about the actual value x is that

$$x \in [\widetilde{x} - \Delta, \widetilde{x} + \Delta].$$

- Expert estimates are also imprecise.

4. Gauging Expert Uncertainty

- Ideally, we should view each expert as a measuring instrument:
  - we compare expert estimates and measurement results, and
  - we get a probability distribution for the estimation error $\Delta x = \widetilde{x} - x$.
- In practice, we rarely have enough samples to make statistically meaningful estimates.
- A reasonable way to describe expert uncertainty is to ask the expert to estimate,
  - for each possible value $x \approx \widetilde{x}$,
  - to what extent x is possible.

5. Gauging Expert Uncertainty (cont-d)

- For example, we can ask the expert to mark her certainty by a mark m on a scale from 0 to s.
- Then we take m/s as the degree.
- The function $\mu(x)$ assigning degree to a value x is known as a *fuzzy set.*
- If for each variable x_i, we only know that $x_i \in \mathbf{x}_i = [\underline{x}_i, \overline{x}_i]$, then we know that

$$y = f(x_1, \ldots, x_n) \in \mathbf{y} = f(\mathbf{x}_1, \ldots, \mathbf{x}_n) \stackrel{\text{def}}{=}$$

$$\{f(x_1, \ldots, x_n) : x_i \in \mathbf{x}_i\}.$$

- Computing such a range $\mathbf{y}$ is one of the main problems of *interval computations.*

6. Processing Expert Uncertainty

- For expert estimates, it is reasonable to consider:
  - for every $\alpha \in [0,1]$,
  - the set $\mathbf{x}_i(\alpha) = \{x_i : \mu_i(x_i) \ge \alpha\}$ of sufficiently possible values.
- Then, for every α, we compute the range

$$\mathbf{y}(\alpha) = f(\mathbf{x}_1(\alpha), \ldots, \mathbf{x}_n(\alpha)).$$

- This can also be done by interval computation techniques.
- Additional problems:
  - sometimes, the dependence $y = f(x_1, \ldots, x_n)$ is not known exactly;
  - even when we know the exact dependence, we can often only compute $f(x_1, \ldots, x_n)$ approximately.

7. Processing Expert Uncertainty (cont-d)

- The approximate character of computing $f(x_1, \ldots, x_n)$ is caused by:
  - rounding errors for arithmetic operations,
  - inevitably imprecise formulas for non-arithmetic elementary functions such as $\exp(x)$ etc.
- One of the main objectives of metrology is:
  - to provide guaranteed information about the actual values of the quantities of interest
  - based on measurement results and expert estimates.

8. Interval and Fuzzy Computations in Metrology: A Brief History

- 1960s: IFIP (led by Wilkinson) proposes:
  - accompanying each data processing software
  - with bounds (interval) estimate of the result's inaccuracy.
- 1960s: Moore et al. proposed general interval techniques for such estimates.
- 1970s: software packages with guaranteed bounds (e.g., Linpack).
- 1965: fuzzy sets introduced by Zadeh.
- 1980s: L. K. Reznik combined expert estimates with measurement intervals in practical problems.
- 1985: first standard for metrological support of data processing.

9. Interval and Fuzzy Computations in Metrology: A Brief History (cont-d)

- 1985: systematic way of providing such support described in a special issue of *Measuring Techniques*.
- 1990s: further theoretical development and algorithms design.
- 2000s–2010s: metrological proposals for taking interval and fuzzy uncertainty into account.
- *What we would like:* to incorporate interval and fuzzy techniques in metrological practice.
- *What is needed for this:*
  - add interval and fuzzy computations to the existing metrological standards,
  - make the corresponding algorithms as simple as possible and as clear to engineers as possible.

10. Case Study: Heat Meter

- In many practical situations, we need to know how much heat or cooling was generated or consumed.
- For example, in nuclear power stations:
  - water or gas is heated by a reactor,
  - the steam is moved to a turbine that generates electricity,
  - when the steam rotates the turbine, it loses energy and cools down.
- Similarly, in heating and air conditioning systems:
  - hot water is circulated, heating a building;
  - cold air is circulated, cooling the building;
  - in dry areas, water is used to cool the buildings.
- In all these cases, it is desirable to measure the amount of heat.

11. Case Study: Heat Meter (cont-d)

- The amount is difficult to measure directly.
- So, heat meters measure flow rate, pressure, in- and out-temperatures and compute the heat flow as

$$\text{flow\_rate}_{\text{out}} \cdot t^\circ_{\text{out}} - \text{flow\_rate}_{\text{in}} \cdot t^\circ_{\text{in}}.$$

- Existing standards only take into account uncertainty in temperature sensors.
- Thus, the existing method underestimate measurement error.
- There is also uncertainty is measuring flow rate.
- Some of this uncertainty comes from inhomogeneity, which needs expert estimates.
- We (K.S. and G.S.) took this into account and got estimates consistent with more accurate measurements.

12. Acknowledgments

This work was supported in part:

- by the National Science Foundation grants:
  - HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and
  - DUE-0926721, and
- by an award from Prudential Foundation.