

**BIWEEKLY PROBLEM NO. 38**

NOVEMBER 1 – 15, 2021

**Problem.** Determine, up to isomorphism, all odd-dimensional unital division algebras over the real numbers.

*Solution.* Let  $A$  be such an algebra,  $a \in A$ , and denote by  $e$  the multiplicative unit of  $A$ . The left-multiplication  $L_a : A \rightarrow A, x \mapsto ax$  is a linear endomorphism on the real vector space  $A$ . Since  $\dim(A)$  is odd,  $L_a$  has a real eigenvalue  $\lambda$  together with an eigenvector  $v \in A$ . Hence  $0 = av - \lambda v = (a - \lambda e)v$ , and since  $A$  is a division algebra, this means  $a = \lambda e$ . Therefore  $A \cong \mathbb{R}$ .