

BIWEEKLY PROBLEM NO. 44

MAR 04 – MAR 18, 2022

Problem. Alice considers a finite simple graph G on $n \geq 3$ vertices. She then assembles a deck of n cards, where each card shows a copy of G with a different vertex (and its incident edges) deleted. Show that if Alice chose a d -regular graph, i.e. if all vertices of G had degree d , then Bob can recognise this and reconstruct G (up to isomorphism) from Alice's deck of cards.

Solution. We first show that Bob can recognise d -regular graphs. Let $G = (V, E)$, and denote by $G_v = (V \setminus \{v\}, E_v)$ the graph with v deleted. Fix any edge in G . This edge will appear on all but 2 of the cards, hence we have $|E| = \frac{1}{n-2} \sum_v |E_v|$. Thus Bob can infer the number of edges in G from examining the cards. By now considering any card G_v , the degree of the absent vertex is $|E| - |E_v|$, and therefore G is d -regular if and only if $d = |E| - |E_v|$ does not depend on v . Notice that since G is simple, any vertex in a G_v has either degree d or degree $d - 1$.

Once this is established, Bob can simply take one of the G_v , draw an extra vertex to it, and draw edges from the new vertex to all of the vertices of degree $d - 1$, thereby obtaining an isomorphic copy of G .