

BIWEEKLY PROBLEM NO. 46

APR 01 – APR 14, 2022

Problem. ¹ Bob states: “I’m precisely of average height, and precisely of average weight. Nonetheless, I’m considered overweight.” – Why is that?

¹This problem was suggested by Jordi-Lluís Figueras.

Solution. Denote the height and weight of a person as random variables H and W , respectively. A scaling argument reveals that $W \approx cH^\alpha$ for constants $c > 0, \alpha > 1$. (Modelling people as balls in space gives $\alpha = 3$, but $\alpha = 2$ turns out to be more accurate, which is why, when calculating a person's BMI, one divides by the square of the height).

Bob being of average height and weight means that his height is $\mathbf{E}[H]$ and his weight is $\mathbf{E}[W]$. However, by Jensen's inequality, we have

$$\mathbf{E}[W] \approx c\mathbf{E}[H^\alpha] \geq c\mathbf{E}[H]^\alpha$$

thus explaining why Bob is considered overweight.