Normalized solutions to polyharmonic equations with Hardy-type potentials via a Nehari–Pohožaev approach

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Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties. This gives rise to the search for *normalized solutions*, i.e., when the $L^2(\mathbb{R}^N)$ norm is prescribed.

In this talk, I will exploit a novel variational approach, introduced in [1] in the context of autonomous Schrödinger equations, to find a least-energy solution to the problem

$$\begin{cases} (-\Delta)^m u + \frac{\mu}{|y|^{2m}} u + \lambda u = g(u), \\ (\lambda, u) \in \mathbb{R} \times H^m(\mathbb{R}^N), \\ \int_{\mathbb{R}^N} u^2 \, \mathrm{d}x = \rho^2, \end{cases}$$

where $\mathbb{R}^N \ni x = (y, z) \in \mathbb{R}^K \times \mathbb{R}^{N-K}$, $N \geq K \geq 2m$, $1 \leq m \in \mathbb{N}$, $\rho > 0$ is given a priori, and the growth of $g\colon \mathbb{R} \to \mathbb{R}$ is mass-supercritical and Sobolev-subcritical at infinity and at least mass-critical at the origin. In particular, λ is part of the unknown and appears as a Lagrange multiplier.

An important step in this approach is to show that all the solutions to the differential equation above satisfy the *Pohožaev identity*, which in the presence of a Hardy-type potential is only known in the case $m = 1$ and $N = K$.

This talk is based on joint work with Bartosz Bieganowski and Jarosław Mederski [2].

References

[1] B. Bieganowski, J. Mederski: *Normalized ground states of the nonlinear Schrödinger equation with at least mass critical growth*, J. Funct. Anal. **280** (2021), no. 11, 108989.

[2] B. Bieganowski, J. Mederski, and J. Schino: *Normalized solutions to at least mass critical problems: singular polyharmonic equations and related curl-curl problems*, arXiv:2212.12361.